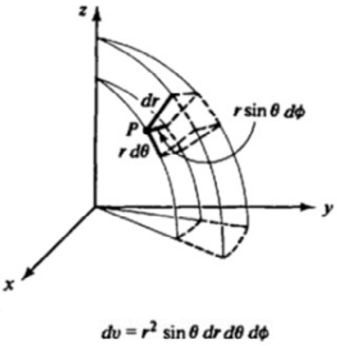
Divergence Theoreme

$$\oint\!\!\oint_S \mathbf{A}\cdot d\mathbf{S} = \iiint_V \nabla\cdot\mathbf{A}\,dV$$



The region r ≤ a in spherical coordinates, it has an electric field

$$\mathbf{E} = \frac{\rho r}{3\varepsilon}\mathbf{a}_r$$

Examine both sides of the divergence theorem for this vector. Consider the surface S as the surface with $r = b \le a$

Solution

On a spherical surface of radius r: $d\mathbf{S} = dS\mathbf{n} = \left(r^2 \sin\theta d\theta d\phi\right)\mathbf{a}_r$

$$\oint\!\!\oint_S \mathbf{E}\cdot d\mathbf{S} = \oint\!\!\oint_S \left(\frac{\rho b}{3\varepsilon}\mathbf{a}_r\right)\cdot\left(b^2 \sin\theta d\theta d\phi \mathbf{a}_r\right) = \int_0^{2\pi}\int_0^{\pi} \frac{\rho b^3}{3\varepsilon}\sin\theta d\theta d\phi = \frac{4\pi\rho b^3}{3\varepsilon}$$

$$dV = r^2 \sin\theta dr d\theta d\phi$$

$$\nabla\cdot\mathbf{E} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\rho r}{3\varepsilon}\right) = \frac{\rho}{\varepsilon}$$

$$\iiint_V (\nabla\cdot\mathbf{E})dV = \int_0^{2\pi}\int_0^{\pi}\int_0^{b} \frac{\rho}{\varepsilon} r^2 \sin\theta dr d\theta d\phi = \frac{4\pi\rho b^3}{3\varepsilon}$$